EXERCISES [MAI 4.5-4.7]

PROBABILITY I (VENN DIAGRAMS – TABLES)

SOLUTIONS

Compiled by: Christos Nikolaidis

A. Paper 1 questions (SHORT)

1. (a)

$n(A)$	15	$n(B)$	25	$n(A\cap B)$	10
$n(A')$	35	$n(B')$	25	$n(A\cup B)$	30
$n(A'\cap B)$	15	$n(A\cap B')$	5	$n(A'\cap B')$	20
$n(A'\cup B)$	45	$n(A\cup B')$	35	$n(A'\cup B')$	40

(b)

$P(A)$	15/50	$P(A')$	35/50	$P(A\cup B)$	30/50
$P(A'\cap B)$	15/50	$P(A'\cup B)$	45/50	$P(B'\cup A)$	35/50

(c)

$P(A\mid B)$	10/25	$P(A'\mid B)$	15/25	$P(B'\mid A)$	5/15
$P(B\mid A)$	10/15	$P(A\mid B')$	5/25	$P(A'\mid B')$	20/25

2. (a)

$P(A)$	0.5	$P(A')$	50	$P(A\cap B)$	0.2
$P(A\cup B)$	0.9	$P(A'\cap B)$	0.4	$P(A'\cup B)$	0.7

(b)

$P(A\mid B)$	1/3	$P(A'\mid B)$	2/3	$P(B'\mid A)$	3/5
$P(B\mid A)$	2/5	$P(A\mid B')$	3/4	$P(A'\mid B')$	1/4

3. (a)

$P(Boy)$	30/80	$P(Group\,C)$	15/80
$P(Boy\ and\ Group\ C)$	10/80	$P(Boy\ or\ Group\ C)$	35/80

(b)

$P(Boy\mid Group\,C)$	10/15	$P(Group\,C\mid Boy)$	10/30
$P(Boy\mid NOT\ Group\,C)$	20/65	$P(NOT\ Group\ C\mid Boy)$	20/30

4. (a)

$P(A)$	$a+c$	$P(A'\cap B)$	b
$P(A')$	$b+d$	$P(A'\cup B)$	$a+c+d$
$P(A\cap B)$	c	$P(A'\cap B')$	d
$P(A\cup B)$	$a+b+c$	$P(A'\cup B')$	$a+b+d$

(b)

$P(A\mid B)$	$c/(b+c)$	$P(B'\mid A)$	$a/(a+c)$
$P(B\mid A)$	$c/(a+c)$	$P(B\mid A')$	$b/(b+d)$
$P(A'\mid B)$	$b/(b+c)$	$P(A'\mid B')$	$d/(a+d)$
$P(A\mid B')$	$a/(a+d)$	$P(B'\mid A')$	$d/(b+d)$

(c) $P(A\cap B)=P(A)P(B)\Rightarrow c=(a+c)(b+c)\Rightarrow a=(2a)(2a)$

$\Rightarrow a=4a^2\Rightarrow a=0.25$, so $b=c=d=0.25$

5. (a) $P(A) = 0.4$ $P(A \cup B) = 0.7$

(b) $x = 0$.

(c) $x = 0.2$

(d) $x = 0.3$

(e) $x = 0.1$

6. (a) $n(A \cup B) = n(A) + n(B) - n(A \cap B)$
$65 = 30 + 50 - n(A \cap B) \Rightarrow n(A \cap B) = 15$ (may be on the diagram)
$n(B \cap A') = 50 - 15 = 35$

(b) $\mathrm{P}(B \cap A') = \dfrac{n(B \cap A')}{n(U)} = \dfrac{35}{100} = 0.35$

7. (a) (i) $n(A \cap B) = 2$

(ii) $\mathrm{P}(A \cap B) = \dfrac{2}{36}\left(\text{or } \dfrac{1}{18}\right)$

(b) $n(A \cap B) \neq 0$ (or equivalent)

8. (a) (i) $n = 0.1$

(ii) $m = 0.2, p = 0.3, q = 0.4$

(b) $\mathrm{P}(B') = 0.6$

9. (a) (i) $p = 0.2$

(ii) $q = 0.4$

(iii) $r = 0.1$

(b) $\mathrm{P}(A \mid B') = \dfrac{2}{3}$

(c) valid reason *e.g.* $\dfrac{2}{3} \neq 0.5$, $0.35 \neq 0.3$ thus, A and B are not independent

10. (a) $p(A \cap B) = 0.6 + 0.8 - 1 = 0.4$

(b) $p(A' \cup B') = p((A \cap B)') = 1 - 0.4 = 0.6$

11. (a) $\dfrac{19}{120}$ $(= 0.158)$

(b) $35 - (8 + 5 + 7)(= 15)$

Probability $= \dfrac{15}{120}\left(= \dfrac{3}{24} = \dfrac{1}{8} = 0.125\right)$

(c) Number studying = 76

Number not studying = 120 – number studying = 44

Probability $= \dfrac{44}{120}\left(= \dfrac{11}{30} = 0.367\right)$

12. (a) (i) $\mathrm{P}(P \mid C) = \dfrac{20}{20+40} = \dfrac{1}{3}$

(ii) $\mathrm{P}(P \mid C') = \dfrac{30}{30+60} = \dfrac{1}{3}$

(b) Investigating conditions, or some relevant calculations

P is independent of C, **with** valid reason

13. (a)

	Boy	Girl	Total
TV	**13**	**25**	**38**
Sport	33	29	**62**
Total	46	**54**	100

$P(TV) = \frac{38}{100}$

(b) $P(TV \mid Boy) = \frac{13}{46}$

14. (a)

	Males	Females	Totals
Unemployed	**20**	**40**	**60**
Employed	**90**	**50**	**140**
Totals	**110**	**90**	200

(b) (i) $P(\text{unemployed female}) = \frac{40}{200} = \frac{1}{5}$

(ii) $P(\text{male I employed person}) = \frac{90}{140} = \frac{9}{14}$

15. (a) 46/97

(b) 13/51

(c) 59/97

16. (a) (i) $P(\text{male or tennis}) = \frac{12}{20} \left(= \frac{3}{5}\right)$

(ii) $P(\text{not football} \mid \text{female}) = \frac{6}{11}$

(b) $P(\text{first not football}) = \frac{11}{20}$, $P(\text{second not football}) = \frac{10}{19}$

$P(\text{neither football}) = \frac{11}{20} \times \frac{10}{19}$

$P(\text{neither football}) = \frac{110}{380} \left(= \frac{11}{38}\right)$

17. (a) Independent (I)
(b) Mutually exclusive (M)
(c) Neither (N)

18. Using a tree diagram,

	ARGENTINE	NOT ARGENTINE	TOTAL
SPANISH	12	3	15
ENGLISH	3	3	6
TOTAL	15	6	21

$p(S \mid A) = \frac{12}{15} = \frac{4}{5}$

19. (a) $P(A) = \frac{1}{11}$

(b) $P(B \mid A) = \frac{2}{10}$

(c) $P(A \cap B) = \frac{1}{11} \times \frac{2}{10} = \frac{2}{110}$

20. (a) $\frac{120}{360}\left(=\frac{1}{3}=0.333\right)$

(b) $\frac{90+120}{360}\left(=\frac{210}{360}=\frac{7}{12}=0.583\right)$

(c) $\frac{90}{210}\left(=\frac{3}{7}=0.429\right)$

21. (a) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$P(A \cap B) = \frac{1}{2}+\frac{3}{4}-\frac{7}{8} = \frac{3}{8}$

(b) $P(A|B) = \frac{P(A \cap B)}{P(B)}\left(=\frac{\frac{3}{8}}{\frac{3}{4}}\right) = \frac{1}{2}$

(c) Yes, the events are independent

EITHER $P(A|B) = P(A)$ **OR** $P(A \cap B) = P(A)P(B)$

22. (a) Independent $\Rightarrow P(A \cap B) = P(A) \times P(B) = 0.3 \times 0.8 = 0.24$

(b) $P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.3 + 0.8 - 0.24 = 0.86$

(c) No, **since** $P(A \cap B) \neq 0$ or $P(A \cup B) \neq P(A) + P(B)$

23. (a) $P(F) = \frac{P(E \cap F)}{P(E)}$, $P(F) = \frac{\frac{1}{3}}{\frac{2}{3}} = \frac{1}{2}$

(b) $P(E \cup F) = P(E) + P(F) - (P(E \cap F)) = \frac{2}{3}+\frac{1}{2}-\frac{1}{3} = \frac{5}{6}(=0.833)$

24. (a) $\frac{3}{4}$

(b) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$P(A \cap B) = P(A) + P(B) - P(A \cup B) = \frac{2}{5}+\frac{3}{4}-\frac{7}{8} = \frac{11}{40}$ (0.275)

(c) $P(A|B) = \frac{P(A \cap B)}{P(B)}\left(=\frac{\frac{11}{40}}{\frac{3}{4}}\right) = \frac{11}{30}$ (0.367)

25. (a) $P(A \cap B) = P(A) \times P(B) = 0.6x$

(b) (i) $P(A \cup B) = P(A) + P(B) - P(A)P(B)$

$\Leftrightarrow 0.80 = 0.6 + x - 0.6x \Leftrightarrow 0.2 = 0.4x \Leftrightarrow x = 0.5$

(ii) $P(A \cap B) = 0.3$

(c) $P(A \cap B) \neq 0$

26. Let $P(A) = x$ then $P(B) = 3x$ and $P(A \cap B) = P(A) \times 3P(A) = 3x^2$

$P(A \cup B) = P(A) + P(B) - P(A \cap B) \Leftrightarrow 0.68 = x + 3x - 3x^2$

$3x^2 - 4x + 0.68 = 0 \Leftrightarrow x = 0.2$ ($x = 1.133$, not possible)

$P(B) = 3x = 0.6$

27. (a) $P(A \cap B) = P(A) + P(B) - P(A \cup B) = \frac{3}{11} + \frac{4}{11} - \frac{6}{11} = \frac{1}{11}$ (0.0909)

(b) For independent events, $P(A \cap B) = P(A) \times P(B) = \frac{3}{11} \times \frac{4}{11} = \frac{12}{121}$ (0.0992)

28.

(a) (i) Use of $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$0.6 = 0.5 + 0.3 - P(A \cap B)$

$P(A \cap B) = 0.2$

(ii) $P(A)P(B) = 0.15 \neq P(A \cap B)$

Hence not independent

(b) Use of $P(B \mid A) = \frac{P(B \cap A)}{P(A)}$

$= \frac{0.2}{0.5}$

$= 0.4$

29. As $P(A \mid B) = P(A)$ then A and B are independent events

Using $P(A \cup B) = P(A) + P(B) - P(A) \times P(B)$

to obtain $0.8 = 0.6 + P(B) - 0.6 \times P(B)$

$0.8 = 0.6 + 0.4P(B) \Rightarrow P(B) = 0.5$ **OR** by using Venn diagram

30.

(a) $P(A \cap B) = P(A) \times P(B \mid A)$

$$= \frac{1}{5} \times \frac{1}{4} \quad \left(= \frac{1}{20}\right)$$

(b) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$$\Rightarrow P(B) = \frac{7}{10} - \frac{1}{5} + \frac{1}{20}$$

$$= \frac{11}{20}$$

(c) **METHOD 1**

$P(B) = \frac{11}{20}$ and $P(B \mid A) = \frac{1}{4}$

$P(B) \neq P(B \mid A)$

$\Rightarrow$ A and B are not independent

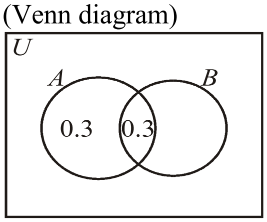
METHOD 2

$P(A) \times P(B) = \frac{1}{5} \times \frac{11}{20} = \frac{11}{100}$ and $P(A \cap B) = \frac{1}{20}$

$P(A \cap B) \neq P(A) \times P(B)$

$\Rightarrow$ A and B are not independent

31. (Venn diagram)



$P(A \cap B) = P(A)P(B)$ $\quad 0.3 = 0.6 \times P(B)$

$P(B) = 0.5$

Therefore, $P(A \cup B) = 0.8$

32. (a) $0.88 = 0.4 + P(B) - 0.4P(B)$

$0.6P(B) = 0.48 => P(B) = 0.8$

(b) $P(A \cup B) - P(A \cap B) = 0.88 - 0.32 = 0.56$

33. for independence $P(A \cap B) = P(A) \times P(B)$

$P(A \cup B) = P(A) + P(B) - P(A) \times P(B)$

$0.52 = P(A) + 2P(A) - 2P(A)P(A)$

$P(B) = 0.4$

34. Total number of possible outcomes = 36

(i) $P(E) = P(1,1) + P(2,2) + P(3,3) + P(4,4) + P(5,5) + P(6,6) = \frac{6}{36}$

(ii) $P(F) = P(6,4) + P(5,5) + P(4,6) = \frac{3}{36}$

(iii) $P(E \cup F) = P(E) + P(F) - P(E \cap F)$

$P(E \cap F) = \frac{1}{36}$

$$P(E \cup F) = \frac{6}{36} + \frac{3}{36} - \frac{1}{36} \quad \left(= \frac{8}{36} = \frac{2}{9}, 0.222 \right)$$

35. Sample space ={(1, 1), (1, 2) ... (6, 5), (6, 6)}

(This may be indicated in other ways, e.g, a grid or a tree diagram, partly or fully completed)

(a) P (S < 8) = 21/36 = 7/12

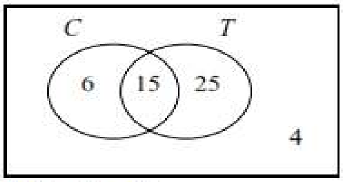
(b) P (at least one 3) = $\frac{11}{36}$

(c) P (at least one 3 | S < 8) = 7/21 = $\frac{1}{3}$

B. Paper 2 questions (LONG)

36.

(a) **METHOD 1**



$\mathrm{P}(T \cap C) = 0.3$

METHOD 2

$n(T \cup C) = n(T) + n(C) - n(T \cap C)$

$46 = 40 + 21 - n(T \cap C)$

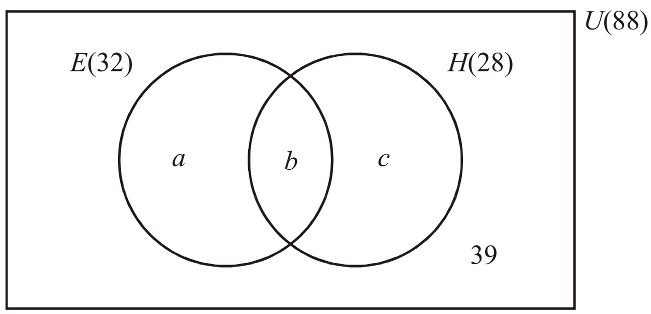
$n(T \cap C) = 15$

$\mathrm{P}(T \cap C) = 0.3$

(b) $\mathrm{P}(T \mid C) = \dfrac{\mathrm{P}(T \cap C)}{\mathrm{P}(C)}$

$= \dfrac{0.3}{0.42}$ $\left(= \dfrac{5}{7}, 0.714 \text{ to 3 s.f.}\right)$

37. (a)



$n(E \cup H) = a + b + c = 88 - 39 = 49$

$n(E \cup H) = 32 + 28 - b = 49 \Leftrightarrow b = 11$

$a = 32 - 11 = 21$
$c = 28 - 11 = 17$

(b) (i) $\mathrm{P}(E \cap H) = \dfrac{11}{88} = \dfrac{1}{8}$

(ii) $\mathrm{P}(H' \mid E) = \dfrac{\mathrm{P}(H' \cap E)}{\mathrm{P}(E)} = \dfrac{\frac{21}{88}}{\frac{32}{88}} = \dfrac{21}{32}$ (= 0.656) Or directly = $\dfrac{21}{32}$

(c) (i) P(none in economics) = $\dfrac{56 \times 55 \times 54}{88 \times 87 \times 86} = 0.253$

(ii) P(at least one) = 1 – 0.253= 0.747
OR
$3\left(\dfrac{32}{88} \times \dfrac{56}{87} \times \dfrac{55}{86}\right) + 3\left(\dfrac{32}{88} \times \dfrac{31}{87} \times \dfrac{56}{86}\right) + \dfrac{32}{88} \times \dfrac{31}{87} \times \dfrac{30}{86} = 0.747$

38. (a) $P(F \cup S) = 1 - 0.14$ (= 0.86)

$P(F \cap S) = 0.93 - 0.86 = 0.07$

Note: You can use *Venn Diagram*

(b) $P(F \mid S)\left(=\frac{P(F \cap S)}{P(S)}\right) = \frac{0.07}{0.62} = 0.113$

(c) F and S are **not** independent

EITHER

If independent $P(F \mid S) = P(F)$, $0.113 \neq 0.31$

OR

If independent $P(F \cap S) = P(F)\,P(S)$, $0.07 \neq 0.31 \times 0.62$ (= 0.1922)

(d) Let $P(F) = x$

$P(S) = 2P(F) = 2x$

$P(F \cup S) = P(F)P(S) - P(F)P(S) \Leftrightarrow 0.86 = x + 2x - 2x^2 \Leftrightarrow 2x^2 - 3x + 0.86 = 0$

$x = 0.386$, $x = 1.11$

$P(F) = 0.386$

39. (a) (i) $P(A) = \frac{80}{210} = \left(\frac{8}{21} = 0.381\right)$

(ii) $P(\text{year 2 art}) = \frac{35}{210} = \left(\frac{1}{6} = 0.167\right)$

(iii) No (the events are not independent

EITHER $P(A \cap B) = P(A) \times P(B)$ (to be independent)

$P(B) = \frac{100}{210}\left(= \frac{10}{21} = 0.476\right)$ but $\frac{1}{6} \neq \frac{8}{21} \times \frac{10}{21}$

OR $P(A) = P(A \mid B)$ (to be independent)

$P(A \mid B) = \frac{35}{100}$ but $\frac{8}{21} \neq \frac{35}{100}$

OR $P(B) = P(B \mid A)$ (to be independent)

$P(B) = \frac{100}{210}\left(= \frac{10}{21} = 0.476\right), P(B \mid A) = \frac{35}{80}$ but $\frac{35}{80} \neq \frac{100}{210}$

(b) $n(\text{history}) = 85$

$P(\text{year 1} \mid \text{history}) = \frac{50}{85} = \left(\frac{10}{17} = 0.588\right)$

(c) $\left(\frac{110}{210} \times \frac{100}{209}\right) + \left(\frac{100}{210} \times \frac{110}{209}\right) \left(= 2 \times \frac{110}{210} \times \frac{100}{209}\right) = \frac{200}{399} (= 0.501)$

40. (a) (i) Venn diagram, 30

(ii) 45

(b) (i) $\frac{70}{100}\left(=\frac{7}{10}\right)$

(ii) $\frac{45}{70}\left(=\frac{9}{14}\right)$

(c) $P(A \cap B) = 0.3 \neq 0$

(d) $P(A \cap B) \neq P(A) \times P(B)$, $\frac{30}{100} \neq \frac{75}{100}\times\frac{55}{100}, \frac{30}{55} \neq \frac{75}{100}$

OR $P(B \mid A) \neq P(B)$, $\frac{30}{55} \neq \frac{75}{100}$

41. (a) (i) $s = 1$

(ii) $q = 5$

(iii) $p = 7, r = 3$

(b) (i) P(art|music) = $\frac{5}{8}$

(ii) **METHOD 1**

$P(\text{art}) = \frac{12}{16}\left(=\frac{3}{4}\right)$ $\frac{3}{4} \neq \frac{5}{8}$ the events are not independent

METHOD 2

P(art) × P(music) = $\frac{96}{256}\left(=\frac{3}{8}\right)$ $\frac{12}{16}\times\frac{8}{16} \neq \frac{5}{16}$ the events are not independent

(c) P(first takes only music) = $\frac{3}{16}$

P(second takes only art)= $\frac{7}{15}$

P(music and art)= $\frac{3}{16}\times\frac{7}{15} = \frac{21}{240}\left(=\frac{7}{80}\right)$

42. (a) (i) 250 (ii) 166 (since 1000/6=166.67) (iii) 83

(b) 250/1000 = 1/4

(c) 166/1000

(d) 83/1000

(e) 250-83 = 167 so P=167/1000.

(f) 250 + 166 – 83 = 333 so P=333/1000

(g) 250 + 166 – 2×83 = 250 so P=250/1000.